Word: convex Word Look it up

Convex

An object is **convex** if for any pair of points within the object, any point on the line that joins them is also within the object. For example, a solid cube is convex, but anything that is hollow or has a dent in it is not convex.

Convex set

In mathematics, convexity can be defined for subsets of any real or complex vector space. Such a subset *C* is said to be **convex** if, for all *x* and *y* in *C* and all *t* in the interval [0,1], the point $tx + (1-t)y$ is in *C*. In words, every point on the straight line segment connecting *x* and *y* is in *C*

The convex subsets of **R** (the set of real numbers) are simply the intervals of **R**. Some examples of convex subsets of Euclidean 3-space are the Archimedean solids and the Platonic solids. The Kepler solids are examples of non-convex sets.

The intersection of any collection of convex sets is itself convex, so the convex subsets of a (real or complex) vector space form a complete lattice. This also means that any subset *A* of the vector space is contained within a smallest convex set (called the convex hull of *A*), namely the intersection of all convex sets containing *A*.

If one restricts to closed convex sets, they can actually be characterised as the intersections of closed half-spaces, lying to one side of a hyperplane. From what has just been said, it is clear that such intersections are convex, and they will also be closed sets. For the converse, one need the *supporting hyperplane theorem* in the form that for a given closed convex set C and point P outside it, there is a closed half-space H that contains C and not P. (That theorem is a case of the Hahn-Banach theorem of functional analysis, but less deep.)

Convex function

A real-valued function *f* defined on an interval (or on any convex subset of some vector space) is called **convex** if for any two points *x* and *y* in its domain and any *t* in [0,1], we have :f(tx+(1-t)y)\\leq t f(x)+(1-t)f(y).

A function is also said to be **strictly convex** if :f(tx+(1-t)y) < t f(x)+(1-t)f(y).

One may compare this definition of convexity and that for sets, and note that a function is convex if, and only if, the region of the plane lying above the graph of said function is a convex set.

Properties of convex functions

A convex function defined on some open set is continuous on the whole interval and differentiable at all but at most countably many points. A twice differentiable function of one variable is convex on an interval if and only if its second derivative is non-negative there and strictly convex if and only if its second derivative is positive; this gives a practical test for convexity.

Any local minimum of a convex function is also a global minimum. A *strictly* convex function will have at most one global minimum.

A convex function respects the Jensen's inequality.

Examples of convex functions

- The second derivative of x^2 is 2; it follows that x^2 is a convex function of *x*.
- The absolute value function $|x|$ is convex, even though it does not have a derivative at $x = 0$.
- The function $f(x) = x$ is convex but not strictly convex.
- The function x^3 has second derivative $6x$; thus it is convex for $x \geqslant 0$ and concave for $x \leqslant 0$.